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Formulas

Ohm’s Law

\[ \text{Rt} = R_1 + R_2 + R_3 \ldots \quad \text{[for series]} \]
\[ Zt = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots} \quad \text{[for parallel]} \]

Capacitive reactance

\[ X = \frac{1}{\pi FC} \quad \text{or} \quad C = \frac{1}{\pi FX} \]

Inductive reactance

\[ X = 2\pi FL \quad \text{or} \quad L = \frac{X}{2\pi F} \]

Reactive impedance

\[ Z = \sqrt{R^2 + (X_l - X_c)^2} \quad \text{[for series circuits]} \]
\[ Z = \frac{R|X_l - X_c|}{\sqrt{R^2|X_l - X_c|^2 + X_l^2X_c^2}} \quad \text{[for parallel circuits]} \]

Resonance

\[ F = \frac{1}{2\pi \sqrt{LC}} \]
Power

RMS: \( \sin 45^\circ (\text{Peak}) \) \hspace{1cm} [\sin 45^\circ = 0.707]

Peak: \( \text{Peak-to-Peak}/2 \)

\[
P = I^2 Z, \quad P = \frac{E^2}{Z} \quad \text{and} \quad P = EI
\]
\[
E = IZ, \quad E = \sqrt{PZ} \quad \text{and} \quad E = \frac{P}{I}
\]
\[
I = \frac{E}{Z}, \quad I = \frac{P}{\sqrt{Z}} \quad \text{and} \quad I = \frac{P}{E}
\]
\[
Z = \frac{E}{I}, \quad Z = \frac{P}{I^2} \quad \text{and} \quad Z = \frac{E^2}{P}
\]

Decibels

Power: \( dB = 10 \log \frac{X}{Y} \)

Voltage: \( dB = 20 \log \frac{X}{Y} \)

Phase

\[
\theta = \operatorname{ArcTan} \left( \frac{X_L - X_C}{R} \right) \quad \text{[for series circuits]}
\]
\[
\theta = \operatorname{ArcTan} \left( \frac{R[X_L - X_C]}{X_L X_C} \right) \quad \text{[for parallel circuits]}
\]
Resistors

Ohm’s Law

Series connection

\[ Rt = R1 + R2 \]

Parallel connection

\[ Rt = \frac{1}{\frac{1}{R1} + \frac{1}{R2}} \]

Examples:

Series R1=8 ohms, R2=8 ohms, Rt=16 ohms
Parallel R1=8 ohms, R2=8 ohms, Rt=4 ohms

For series/parallel, calculate the branches separately.

An example is the L-Pad, which has a series and parallel component connected to the speaker load. So calculate the parallel branch first, to find the equivalent value. Then calculate this value connected in series with the other resistance to find the total.

Speaker=8 ohms, Series resistance = 4 ohms, Parallel resistance=6 ohms

Parallel branch = 3.43 ohms, Rt=7.43 ohms

Voltage Dividers

A pair of resistors forms a voltage divider, where voltage (and power) is proportioned within the system. A fixed ratio is produced, so the value at the load is a percentage of the input. This can be expressed in decibels.

The simplest voltage divider is one where two resistors of the same value are connected in series. This always results in an even division of voltage and power between the two. Such an arrangement always results in 6dB reduction.
An example is a simple network having two 8 ohm resistors. If a speaker motor were a perfectly resistive 8 ohm load, then R2 would represent the loudspeaker load. R1 could be a series resistor used as an attenuator, or it could also be another loudspeaker motor. At any rate, this simplified model of a purely resistive loudspeaker motor is how speaker circuits are often visualized.

![Diagram of a resistive network]

This connection ensures that the same current will pass through each series component.

Find current through the network, using a reference voltage:

\[ I = \frac{E}{R} \]

Using 10 volts as our reference, we see that current would be equal to 10/16 or 0.625 amperes. This can also be written as 0.625A or 625mA.

We can find the voltage across each component by rearranging the formula:

\[ E = IR \]

Since 625mA passes through the series circuit, we find that the voltage across each resistor is 0.625 x 8, or 5 volts. This makes sense, that the 10 volts across the network would be split 50/50 across equal value resistors.

Now to find the ratio expressed in decibels:

\[ dB = 20 \log \frac{X}{Y}; \quad 6 = 20 \log \left(\frac{5}{10}\right) \]

So we can always expect 6dB reduction from a series resistance equal to the load.
Another example is a network having a 25 ohm resistor and an 8 ohm resistor connected in series. This would be a common configuration for a circuit where R1 were a series attenuator and R2 were a loudspeaker motor.

Total resistance is \(25 + 8 = 33\) ohms. So find current through the network, using a reference voltage:

\[ I = \frac{E}{R} \]

Using 10 volts as our reference, we see that current would be equal to \(10/33\) or 0.303A, which is also written as 303mA.

We can find the voltage across each component by rearranging the formula:

\[ E = IR \]

Since 303mA passes through the series circuit, we find that the voltage across R1 is \(0.303 \times 25 = 7.58\)v and R2 is \(0.303 \times 8 = 2.42\)v. Notice that the two voltages add up to equal the source, which is exactly what we might expect. This is true of two purely resistive components connected in series.

Now to find the amount of attenuation to R2 expressed in decibels:

\[ dB = 20\log X/Y; \quad 12.3 = 20 \log (2.42/10) \]

So this circuit provides 12dB attenuation. Interestingly, to double the amount of attenuation required much more than double the amount of series resistance.
Here’s a simple parallel network having two 8 ohm resistors. In this case, the most likely reason for the connection would be to use two loudspeakers together on a common line. Another reason to make this sort of connection is to add an 8 ohm resistor across a speaker for impedance matching reasons or to change system damping, which will be discussed later in this document.

*This connection ensures that the same voltage will be across both parallel components.*

Find current through each component, using a reference voltage:

\[ I = \frac{E}{R} \]

Using 10 volts as our reference, we see that current would be equal to 10/8 or 1.25A through each individual 8 ohm resistor. Also, we see that since the total resistance of the network is equal to 4 ohms, the total current through the network is 2.5A with a 10 volt source.
Another form of voltage divider is one that has a series resistance and a parallel value in shunt across a load. In fact, this is the most common form of voltage divider, where the load has higher impedance than the divider and doesn’t modify its behavior very much.

This network is also known as an *L-Pad* and its advantages are that the source impedance is reduced and load damping is increased.

First, find the resistance of R2 and R3 in parallel, $1 / (1/2.7 + 1/8) = 2$ ohms. That means that the total resistance across the circuit is 8 ohms, since this 2 ohms is added to the value of R1. Again, we can find current through the network, using a reference voltage:

$I = E/R$

Using 10 volts as our reference, we see that current would be equal to $10/8$ or 1.25A.

We can find the voltage across each component by rearranging the formula:

$E = IR$

Since 1.25A passes through the series circuit, we find that the voltage across R1 is $1.25 \times 6 = 7.5$V. But don’t calculate $1.25A \times 8$ for R3 or $1.25A \times 2.7$ for R2 because the current is split between them and voltage is the same across them. Common sense tells us that the voltage across the parallel components R2 and R3 is equal to $10 - 7.5$ (the voltage across R1) and this common sense would be right, at least for purely resistive circuits. But to calculate using Ohm’s law, we must use the combined value of R2 and R3 in parallel – 2 ohms – and calculate this with our total current, 1.25A. So $1.25A \times 2 = 2.5$V, just as we expected. We can then calculate the current through each parallel leg if we wish, by using the formula $I = E/R$. But since we know that the voltage across R2 and R3 is 2.5 volts, we can directly calculate the attenuation in decibels:

$dB = 20 \log X/Y; \quad 12 = 20 \log (2.5/10)$

*So this circuit provides 12dB attenuation.* Notice that attenuation is the same as a single 25 ohm resistor, but the source impedance of the L-Pad is 8 ohms and the source impedance of the 12dB series attenuator is 33 ohms. More importantly, the damping of component R3 is increased, which is important if R3 is reactive, as speaker motors usually are.
Reactive components

Reactive impedance

Inductors

\[ X = 2\pi FL, \text{ Rearranged to find for Inductance, } L = \frac{X}{2\pi F} \]

Capacitors

\[ X = \frac{1}{2\pi FC}, \text{ Rearranged to find for Capacitance, } C = \frac{1}{2\pi FX} \]

*X is reactive impedance, measured in ohms.* Reactive impedance is like resistance – It is an impedance to current and can be calculated with Ohm’s law, same as resistors. *But only with other components of the same type.* In other words, if only inductors are in a circuit, then Ohm’s law applies. If only capacitors are in a circuit, then Ohm’s law applies again. But if reactive components are mixed, or if resistance is included, then other calculations must be used.

This is because capacitance has voltage leading current, and is 90 degrees out of phase with resistance. Inductance has current leading resistance and is 90 degrees out of phase the other way. So inductance and capacitance are 180 degrees away from each other, and as you might expect, this brings some unusual and interesting properties. One of them is *resonance*, which is discussed later.

Another interesting property is the relationship of frequency to impedance with reactive components. That is one of their most important features. A reactive component – inductor or capacitor – has specific impedance at only one frequency. As frequency rises, inductive impedance increases but capacitive impedance decreases. Said another way, an inductor’s impedance rises as frequency goes up and a capacitor’s impedance falls as frequency goes up. So if a capacitor is 16 ohms at 2kHz, it will be 8 ohms at 4kHz, and so on.

Examples:

10\(\mu\)F capacitor

At 100Hz \[ X = \frac{1}{2\pi FC}, \text{ } X = \frac{1}{(2 \pi (100) (10^{-6}))}, \text{ } X = 159 \text{ ohms} \]
At 1kHz \[ X = \frac{1}{2\pi FC}, \text{ } X = \frac{1}{(2 \pi (1000) (10^{-6}))}, \text{ } X = 15.9 \text{ ohms} \]
At 10kHz \[ X = \frac{1}{2\pi FC}, \text{ } X = \frac{1}{(2 \pi (10000) (10^{-6}))}, \text{ } X = 1.59 \text{ ohms} \]

2mH inductor

At 100Hz \[ X = 2\pi FL, \text{ } X = 2 \pi (100) (2 \times 10^{-3}), \text{ } X = 1.25 \text{ ohms} \]
At 1kHz \[ X = 2\pi FL, \text{ } X = 2 \pi (1000) (2 \times 10^{-3}), \text{ } X = 12.5 \text{ ohms} \]
At 10kHz \[ X = 2\pi FL, \text{ } X = 2 \pi (10000) (2 \times 10^{-3}), \text{ } X = 125 \text{ ohms} \]
Voltage divider with reactive components

The simplest circuit with reactive components is one having only a single type. An example would be a couple of coils connected in series. In this kind of circuit, the impedance of the circuit changes as a function of frequency, but the proportion of signal division remains constant. For this reason, a coil in series with another coil does not form a filter, not even a first-order filter. It simply forms an attenuator, just like a pair of series resistors would.

An example is a simple network having two 1mH inductors. Like the series resistor arrangement, this connection ensures that the same current will pass through each series component. But unlike the resistor circuit, the current through the circuit will change with respect to frequency.

Find impedance of each coil, using a reference voltage and frequency:

\[ X = 2\pi FL \]

find \( X \) at 1kHz

\[ X = 2 \pi (1000) (1 \times 10^{-3}), \quad X = 6.28 \text{ ohms} \]

So two in series will be 12.56 ohms at 1kHz. The rest of the analysis of this circuit is much the same as is done to calculate for resistors. But in this case, frequency is relevant, because as frequency rises, impedance rises too.

In the case where two series inductors have the same value, you can see that they will proportion voltage across them equally, just like a pair of series resistors. Impedance changes with respect to frequency, but the change is equal in each component, so the voltage division is the same between them no matter what the frequency is.

*So we can always expect 6dB reduction from series inductance equal to load inductance.*
Another example is a network having a 3.3mH inductor and a 1mH inductor connected in series.

Find impedance of each coil, using a reference voltage and frequency:

\[ X = 2\pi FL, \text{ find } X \text{ at } 1\text{kHz} \]

For L1, \[ X = 2 \pi (1000) (3.3 \times 10^{-3}), X = 20.73 \text{ ohms} \]
For L2, \[ X = 2 \pi (1000) (1 \times 10^{-3}), X = 6.28 \text{ ohms} \]

Total impedance is \[ 20.73 + 6.28 = 27.01 \text{ ohms} \]. So find current through the network, using a reference voltage at 1kHz:

\[ I = E/R \]

Using 10 volts as our reference, we see that current would be equal to \[ 10/27 = 0.37 \text{A}, \text{ which is also written as } 370\text{mA}. \]

We can find the voltage across each component by rearranging the formula:

\[ E = IR \]

Since 370mA passes through the series circuit, we find that the voltage across L1 is \[ 0.37 \times 20.73 = 7.68\text{v} \] and L2 is \[ 0.37 \times 6.28 = 2.32\text{v} \]. Notice that the two voltages add up to equal the source, which is exactly what we might expect. This is true of two components of the same reactive type connected in series.

Now to find the amount of attenuation to L2 expressed in decibels:

\[ dB = 20\log(X/Y); \ 12.7 = 20 \log (2.32/10) \]

So this circuit provides 12dB attenuation. Interestingly, to double the amount of attenuation required much more than double the amount of series inductance.
Reactive circuits with complex impedance

Most circuits will have reactive components of more than one type. Only the simplest circuits will contain only pure resistance or pure reactance, and such a circuit wouldn’t be particularly useful. Inside our electronic devices are coupling capacitors, bias resistors, bypass capacitors and transformers, all of which are used for a specific purpose and all of which introduce their own reactive nature. So almost all electronic devices contain circuits having complex impedance, and which form filters.

One of the most common is the simple first-order filter. Low-pass filters commonly use an inductor in series with a resistive load and high-pass filters often use a capacitor in series with a resistive load. So a good example to examine would be a first-order high-pass filter, which might be used for a coupling stage in an amplifier or a crossover for a tweeter.

\[
\begin{align*}
\text{C1} & \quad 1 \quad 2 \\
10\text{uF} & \\
8\text{ ohms} & \quad R1 \\
\end{align*}
\]

In this example, the 10uF capacitor C1 will decrease impedance as frequency rises, so more power will be delivered to the 8 ohm load resistor, R1.

To find the value where there is equal division between C1 and R1, we can use the reactive formula for capacitors, and find the frequency where \( X = R1 \).

\[
X = \frac{1}{2\pi FC}, \text{ Rearranged to find for Frequency, } F = \frac{1}{2\pi XC}
\]

\[
F = \frac{1}{2 \pi (8) (10 \text{ E-6})}, F = 1989\text{Hz}, \text{ roughly } 2\text{kHz}.
\]

Note: Since components are manufactured with a tolerance, there is always some ambiguity in these kinds of calculations. One can measure a device and remove this ambiguity, but when getting a part off the shelf it is important to understand that its value will be as stated, plus or minus some tolerance value. For example, when you get a 10K ohm resistor with 10% tolerance, you can expect it to be between 9K and 11K. The tolerance value is stated in manufacturers documentation. So when discussing the frequency where resistance equals capacitive reactance in the example above, it is more realistic to understand that there will be a tolerance of a couple hundred Hertz if the values aren’t known to be specifically as stated.
At approximately 2kHz, the impedance of the 10uF capacitor C1 will be 8 ohms, which is equal to the load resistor R1. One might expect voltage across each component to be \( \frac{1}{2} \) the source value, as it was with resistors. But in the case of circuits with complex reactance, something unusual happens.

The reason is that series impedance is calculated with Pythagorean’s formula, and the total impedance is less than the sum of the two components. So calculate the total impedance of the circuit:

\[
Z = \sqrt{R^2 + |Xl - Xc|^2}
\]

Since there is no inductance in this circuit, “\( Xl \)” = 0, and the formula becomes:

\[
Z = \sqrt{R^2 + Xc^2}, \quad Z = \sqrt{8^2 + 8^2}, \quad Z = \sqrt{128}, \quad Z = 11.31 \text{ ohms}
\]

This is interesting. Notice that we now have two series impedances, each of 8 ohms, but the total impedance is not 16 ohms.

So find current through the network, using a reference voltage at 2kHz:

\[
I = \frac{E}{Z}
\]

Using 10 volts as our reference, we see that current would be equal to \( \frac{10}{11.31} \) or 0.884A, which is also written as 884mA.

Since 884mA passes through the series circuit, we find that the voltage across C1 is 0.884 x 8 = 7.07v and R2 is 0.884 x 8 = 7.07v. Notice that the two voltages do not add up to equal the source, which is the result of having two components of the different reactive type connected in series.

A couple points of interest, or maybe “trivial trivia.” Get out your calculators.

Notice that the voltage across each component was 7.07v. It was 0.707 x the total voltage across the network. This is because the two reactive impedances were equal, resulting in 45 degrees of phase shift. Now, using your calculator, find the SIN of 45 degrees. It’s 0.707. This is a sort of “magic number” in electronics, sort of like \( \pi \). You’ll see this value over and over again.

Now to find the amount of attenuation to R1 expressed in decibels:

\[
\text{dB} = 20 \log X/Y; \quad 3 = 20 \log (7.07/10)
\]

So this circuit provides 3dB attenuation at the frequency where capacitive reactance equals resistance. It also provides 6dB/octave attenuation below that, as is shown in the response chart below. You can calculate a series of points just like we’ve done above to plot this curve:
First-order response curve

The response curve above represents the power developed across the load – resistor R1 in the circuit above – in a first-order high-pass network. The filter shown above has capacitive reactance equal to resistance at 2kHz, so if it were used as a crossover, it would be said to have a crossover frequency of 2kHz. The crossover frequency has 3dB attenuation, and there is 6dB attenuation per octave below that.

Low-pass first order networks are very similar, except the reactive component is an inductor. The curve has the same asymptotic slope but it falls from left to right, passing more energy at low frequencies.
**Resonance**

You may have noticed a peculiar property of reactive circuits is that the sum of all the voltages within the circuit seem to be greater than the source. But wait until you see what happens to a circuit in resonance.

The first thing of interest is the resonant frequency, which is found by the formula:

\[ F = \frac{1}{2\pi \sqrt{LC}} \]

This tells us the precise frequency where inductive reactance and capacitive reactance are equal.

\[ F = \frac{1}{2\pi \sqrt{(0.6E - 3)(10E - 6)}}, \quad F = \frac{1}{2\pi \sqrt{6E - 9}}, \quad F = \frac{1}{4.87E - 4}, \quad F = 2054\text{Hz} \]

So this circuit is in resonance at 2kHz.

From our last example, we saw that the 10uF capacitor C1 was 8 ohms at 2kHz, so it must be very close to this value at 2.054kHz. And the value of the 0.6mH coil L1 must also be very nearly 8 ohms, since resonance requires that inductive reactance be equal to capacitive reactance. But it can’t hurt to run the numbers and see.
\[ X_L = 2\pi FL \text{ and } X_C = \frac{1}{2\pi FC} \]

The resonant frequency is 2054, so these \(X_L\) and \(X_C\) at this frequency:

\[ X_L = 2\pi FL, \quad X_L = 2\pi(2054)(0.6\times10^{-3}), \quad X_L = 2\pi(1.23), \quad X_L = 7.74 \text{ ohms.} \]

\[ X_C = \frac{1}{\pi FC}, \quad X_C = \frac{1}{2\pi(2054)(10E - 6)}, \quad X_C = \frac{1}{2\pi(2.054E - 2)}, \quad X_C = 7.74 \text{ ohms.} \]

Now here’s where it gets interesting. Let’s use a reference voltage and find the current through the circuit. First, we must calculate the total impedance of the circuit:

\[ Z = \sqrt{R^2 + |X_l - X_c|^2} \]

Since there is no resistor in this circuit, “\(R\)” = 0, and the formula becomes:

\[ Z = \sqrt{|X_l - X_c|^2}, \quad Z = \sqrt{7.74 - 7.74}^2, \quad Z = 0 \]

This is really interesting. What this means is that at the resonant frequency, impedance approaches zero so current approaches infinity. No matter what voltage we plug into the formula, \(I = E/Z\), current will be infinite if impedance is zero.

Notice that I used the phrases “approaches zero” and “approaches infinity.” This is because, in practice, there is always some internal resistance in the circuit, and nothing is purely reactive. Even if the circuit is made using superconductors that have ultra-low internal resistance, there still is some. There is resistance in the coil. There is resistance in the source supply, an output transistor or whatever. There is resistance in the connection wires and there is resistance across the dielectric of the capacitor and in its leads. So we’ll not be quite able to get infinite current and power from AA batteries, although the circuit in resonance will certainly cause a shorted condition at this frequency.

This does cause some interesting conditions though. Since the circuit is nearly a short at this frequency, current is only limited by the internal resistance of the circuit. So assuming that the circuit is capable of flowing 10 amperes, the voltage across each component would rise to 7.74 x 10, or 77.4 volts. *This would be true no matter what the source voltage was.*
Here’s an example of the resonant circuit we just discussed. It is a model of a hypothetical circuit, containing only a series 0.6mH inductor and a 10μF capacitor, with no resistance. The source only provides 1.0 volt, but as you can see, the voltage across the coil rises to approach infinity at 2kHz. This is also true of the capacitor. And since the voltage is infinite across the components, so too is the current and the power.
A more reasonable condition can be represented by placing a small value of series resistance in the circuit. In the response curve below, there is 1 ohm of series resistance.
Notice that even with an ohm of series resistance, we still find that eight times as much voltage is across each reactive component in resonance than was applied to the circuit. This circuit has 1 volt applied to it but at resonance there’s 8 volts across each component.

This is why it is important to consider the effects of LC peaking. This condition causes increased energy across reactive components when in resonance. This means that speakers are delivered more power when something in the circuit causes this condition. It can make a peak in the response curve and it can damage speaker motors and crossover components if high volumes are applied and the condition is severe.

Now that we’ve seen how resonance works, let’s examine the effects of peaking in a typical loudspeaker circuit. The voice coil of a tweeter is much like the circuit we just described, but inductance is smaller and internal resistance is higher. Woofers often have much higher inductance, and sometimes peaking in them is huge, often giving a large midbass or midrange peak. But let’s focus on a tweeter, and use simplified but realistic values.

The tweeter’s voice coil is represented by L1 and R1 and the crossover capacitor is C1.
As you can see, the circuit is clearly peaking. This condition is also called *underdamped*, and it looks the same as a speaker box that has *high Q*, which is another way of describing the condition. There is considerably more energy is delivered to the tweeter than we would expect from a pure first-order filter. If we are hoping for 2kHz crossover – such as we might expect from a 10uF crossover capacitor – then this sort of response is not what we want. It has full output at 2kHz and is +3dB from 3kHz to 4kHz.

The way to solve this is with additional *damping*. A simple shunt resistance is usually all that’s required for tweeters, and a Zobel for midrange or woofers is best. Sometimes, this is combined with response modifiers, such as top octave compensation for compression horns.
The simple addition of a single resistor damps the resonant peak and makes the first-order crossover filter more pure. This in turn gives a much better response curve. As you can see, filter peaking has been reduced from 3dB to less than 1dB.
The \textit{Re/Le} model gives us a better picture of what is happening in a loudspeaker circuit than viewing the speaker as if it were a pure resistor, but it is still a simplification. There is another resonance created by the diaphragm and suspension, and if it is used in a horn or bass-reflex cabinet, there are additional resonances as well. Further, the primary voice coil resistance and inductance are nonlinear, and they change with respect to frequency and current.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\caption{Diagram showing the \textit{Re/Le} model and additional resonators.}
\end{figure}

A better model includes the resonant frequency of the diaphragm, as shown above. When modeling horns, it is a good idea to add two or three series resonators.
When using a model of a compression horn that includes its resonances, the response curve shows them as below:

Notice the 4dB peak in the midrange. This is a more accurate picture of what is being delivered to the compression horn, because the model used is more accurate.
And now, here is the response with a 20 ohm damper resistor installed across the tweeter:

![Graph with 20 ohm damper resistor](image1)

Or with an 8 ohm damper resistor:

![Graph with 8 ohm damper resistor](image2)

As you can see, installing a damping resistor of 10-20 ohms provides better response than a single-capacitor crossover without the damper. *This is almost always true.*
Now let’s examine crossover response with a 25 \( \Omega \) resistor for attenuation and with top-octave compensation.

Notice how peaking is increased when this kind of attenuator is used. Since the unity level is now 98dB, it is disturbing that output in the lower midrange – below expected cutoff - is 103dB. You can notice the tweeter having abnormally high output in the lower midrange, and in fact, it sounds louder than 98dB/W/m because of this peak.
We can use another form of compensation network that not only attenuates midrange and augments high frequencies but also damps the resonant peak. We can use series and parallel resistances to provide attenuation. The voltage divider then becomes fundamentally between the two fixed resistors instead of being between the single series attenuator and the tweeter.

So to effect the same 10dB attenuation, we might choose to replace the single series 15 Ω resistor with a series/parallel divider network of 5.5 Ω and 3.7 Ω. For compensation of the top-octave, a 5μF capacitor is installed across the 5.5Ω series resistor.

This is a good curve for the tweeter circuit, and one that offers substantial performance benefits compared with an uncompensated design.
Now that we’ve seen what kinds of things the crossover can do, let’s look at the actual response of a compression horn. That way we’ll have some idea what to expect from the system as a whole.

First, a look at the response curve of the horn with no crossover at all, to set a “baseline." The lowest frequency shown on these graphs is 300Hz, represented by the leftmost edge. Amplitude response is the dark black curve and phase is shown in gray. Notice the movement in phase, particularly at low frequencies within a couple octaves of cutoff. Both amplitude and phase look very much like a system in resonance at frequencies near the flare rate. This is characteristic of all horns, and happens without the addition of any electrical crossover components at all.

Compression horn without crossover or any electronic components
Now let's put a single 10μF capacitor in series. No other parts.

Compression horn with 10μF capacitor

Interestingly, there is significant energy below 2kHz, and the system is obviously peaking around 500Hz. See the increased output in this region? Notice also that this is below the flare rate of the horn. There is also a dip in response at 2kHz when the circuit is configured this way.

This clearly shows the peaks described by the Spice model. The most obvious one is observed at 500Hz.
Now, let’s look at measured response after adding a damper resistor.

Compression horn with 10uF capacitor and 20 ohm damping resistor

Here again, we see the Spice model confirmed by measured performance. The amount of peaking at 500Hz is noticeably reduced.
If you attenuation the tweeter, the amount of peaking goes back up. In this case, the lower frequencies are now higher than midband and above top octave output.

Compression horn with 10uF capacitor and 10dB attenuator
But the addition of a damper brings it back down.

Compression horn with 10uF capacitor, 10dB attenuator and damper
It is clear that one can obtain good results with this crossover configuration, but it is still important to understand what is going on here. The crossover capacitor is not doing what a person might expect it to do - A 10uF capacitor on compression horn with advertised impedance of 8 ohms is most definitely not acting as though it had 2kHz crossover. The horn is generating output fully two octaves lower. One could install a smaller value capacitor easily enough to raise the crossover frequency as desired. But another solution is to use a higher-order crossover.

This is the response curve of the same compression driver on the same horn, crossed over with a $\pi$ crossover, having a third-order filter with damping and top-octave compensation.
Compression horns aren’t the only subsystems that are vulnerable to LC peaking. Look at the response below, shown of a woofer circuit that has obvious peaking. The green curve shows the response of the tweeter circuit, which is a $\pi$ crossover having a third-order filter with damping and top-octave compensation for compression horns.

**Woofer circuit peaking from voice coil and crossover interaction**

As you can see, woofer peaking with a standard second order network is unacceptably high at almost 20dB. This system will need to have a Zobel damper installed.
The way to fix response of a midrange or woofer circuit that is peaking is to add a Zobel network, which is a form of RC damper.

The formula for calculating an optimal Zobel RC damper is:

\[ C_z = \frac{L_e}{R_e^2} \]
\[ R_z = 1.25R_e \]

So let’s insert an RC damper and investigate system performance.

Response with HF compensation for tweeter and RC damper for woofer
π Speakers

“Crossover Electronics 101”

Circuits and Response Graphs
First-order single capacitor high-pass crossover

Exhibit (1)
First-order high-pass capacitor with damping resistor

Exhibit (2)
“First-order” high-pass crossover with series attenuation resistor

Exhibit (3)
"First-order" high-pass crossover with attenuator and damper

Exhibit (4)

Notice that the response curve now takes on a much steeper slope than 6dB/octave. This is because of voice coil and crossover resonance. Response is flat because filter is properly damped, but slope is greater than 6dB/octave.
“First-order” high-pass crossover with damper, attenuator and HF bypass capacitor

Exhibit (5)
“First-order” crossover with attenuator and HF bypass cap but no damper

Exhibit (6)
Third-order high-pass crossover

Exhibit (7)

*Notice that without any attenuation resistors or L-Pad’s, a third-order filter provides flatter amplitude response than a first-order filter because peaking is minimized.*
Third-order high-pass crossover with attenuation resistor

Exhibit (8)

Notice that once an attenuation resistor is added to this circuit, it becomes extremely underdamped. This configuration is very sensitive to load resistance.
Third-order high-pass crossover with attenuator and damper

Exhibit (9)

Notice damper resistance is in a slightly different configuration than first-order. This is to provide specific damping. An L-Pad also provides specific damping, usually 8Ω or some other common value. In this case, we are targeting a specific transfer function that is not the same as what an 8Ω load would provide.
Third-order high-pass crossover with damper, attenuator and HF bypass capacitor

Exhibit (10)

The HF bypass capacitor provides a little more output in the top-octave because it reduces the attenuation at high frequencies.
Third-order high-pass crossover with attenuator and HF bypass cap, but no damper

Exhibit (11)

Removing the damper resistor makes this circuit highly underdamped, creating the tell-tale resonant peak near the crossover frequency.
Notice that the ratio of $R_1/R_2$ sets peaking for a specific amount that raises the amplitude of the crossover point slightly. This is so that the first two octaves have flat response before HF augmentation starts.
First-order single coil low-pass crossover

Exhibit (13)

Notice that the coil doesn’t really provide as much a crossover as it makes a flat attenuation above the midrange band. This is because the woofer is fundamentally inductive and forms a voltage divider with the “crossover” coil.
Second-order low-pass crossover

Exhibit (14)

Woofer $L_e=1.5\text{mH}$, $R_e=6.0\text{ ohms}$, $F_{ts}=45\text{Hz}$, $Q_{ms}=7$, $Q_{es}=0.4$, $Q_{ts}=0.38$

Notice that the filter is peaking quite a bit. This is because the crossover and voice coil are in a resonant condition.
Second-order low-pass crossover with Zobel damper

Exhibit (15)

\[ R_z = 8 \text{ ohms}, \quad C_z = 30\mu F \]
I have found that varying Zobel capacitor values doesn’t change the woofer response very much. But it does affect phase enough to change the interaction with the adjacent driver slightly. So careful manipulation of the Zobel capacitor can help the designer “dial-in” the position of the forward lobe.